Dr- Mathematics

Topic- Partial Differential Equation

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(1) General method for solving Partial differential equation with two independent Variables.

Let the general Partial differential equation of first order is represented by

$$F(x, y, z, p, q) = 0 \quad \text{---(1)}$$

where x, y are independent Variables and z is dependent Variable and

$$p = \frac{\partial z}{\partial x} \quad \text{and} \quad q = \frac{\partial z}{\partial y}$$

Since z depends on x, y, we have

$$dz = \frac{\partial z}{\partial x} dx + \frac{\partial z}{\partial y} dy \quad \text{---(2)}$$

i.e. $dz = p\,dx + q\,dy$ ---(2)

The main thing in charpit's method is to find another relation between the Variables x, y, z and the Partial diff. coefficient such that when the Values of p and q obtained by solving (1)

and desired equations are substituted in (2), it becomes integrable. The integration of (2) will give the complete Integral of (1)

Let us assume that such a relation is given by the equation $(x, y, z, p, q) = 0$ — (3)

The values of p and q thus obtained when substituted in (2) will render it integrable

In order to obtain (4), we differentiate (1) & (3) w.r.t x & y.

Then we get the following equations

$$\left.\begin{aligned} \frac{\partial F}{\partial x} + \frac{\partial F}{\partial z} p + \frac{\partial F}{\partial p} \cdot \frac{\partial p}{\partial x} + \frac{\partial F}{\partial q} \cdot \frac{\partial q}{\partial x} &= 0 \\ \frac{\partial f}{\partial x} + \frac{\partial f}{\partial z} p + \frac{\partial f}{\partial p} \cdot \frac{\partial p}{\partial x} + \frac{\partial f}{\partial q} \cdot \frac{\partial q}{\partial x} &= 0 \end{aligned}\right\} \quad (4)$$

$$\left.\begin{aligned} \frac{\partial F}{\partial y} + \frac{\partial F}{\partial z} \cdot q + \frac{\partial F}{\partial p} \cdot \frac{\partial p}{\partial y} + \frac{\partial F}{\partial q} \cdot \frac{\partial q}{\partial y} &= 0 \\ \frac{\partial f}{\partial y} + \frac{\partial f}{\partial z} \cdot q + \frac{\partial f}{\partial p} \cdot \frac{\partial p}{\partial y} + \frac{\partial f}{\partial q} \cdot \frac{\partial q}{\partial y} &= 0 \end{aligned}\right\} \quad (5)$$

Next day.